IMPLICIT DIFFERENTIATION HANDOUT

For # 1-3, find $\frac{dy}{dx}$ by implicit differentiation

1. $x^3 y^3 - y = x$

2. $\text{Sin}x = x(1 + \tan y)$

3. $\text{Cos}\, x^2 = x e^y$

For # 4-5, evaluate the derivative at the indicated point and find the tangent line

4. $y^2 = \frac{x^2 - 9}{x^2 + 9}$, (3, 1)

5. $x^{2/3} + y^{2/3} = 5$, (1,8)

For # 6-7, find $\frac{d^2y}{dx^2}$ in terms of x and y

6. $y^2 = x^3$

7. $\ln(x+y) = x$

SOLUTIONS

1. $\mathbf{x^3 y^3 - y = x}$

$$x^3 [3y^2 \frac{dy}{dx}] + y^3 [3x^2] - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} + 3 x^2y^3 - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$

$$\frac{dy}{dx} (3x^3y^2 - 1) = 1 - 3 x^2y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

2. $\mathbf{\sin x = x (1 + \tan y)}$

$$\cos x = x[\sec^2 y \frac{dy}{dx}] + (1 + \tan y)$$

$$\cos x = x \sec^2 y \frac{dy}{dx} + 1 + \tan y$$

$$\cos x - 1 - \tan y = x \sec^2 y \frac{dy}{dx}$$

$$\frac{\cos x - 1 - \tan y}{x \sec^2 y} = \frac{dy}{dx}$$

3. $\mathbf{\cos x^2 = x e^y}$

$$- \sin x^2 (2x) = x [e^y \frac{dy}{dx}] + e^y$$

$$- 2x \sin x^2 = x e^y \frac{dy}{dx} + e^y$$

$$- 2x \sin x^2 - e^y = xe^y \frac{dy}{dx}$$

$$\frac{-2x \sin x^2 - e^y}{xe^y} = \frac{dy}{dx}$$

4. $\mathbf{y^2 = \frac{x^2 - 9}{x^2 + 9}, (3, 1)}$

$$2y \frac{dy}{dx} = \frac{(x^2 + 9) [2x] - (x^2 - 9) [2x]}{(x^2+9)^2}$$

$$2y \frac{dy}{dx} = \frac{2x^3 + 18x - 2x^3 + 18x}{(x^2+9)^2}$$

$$2y \frac{dy}{dx} = \frac{36x}{(x^2+9)^2}$$

$$\frac{dy}{dx} = \frac{1}{2y} \cdot \frac{36x}{(x^2+9)^2}$$

$$\frac{dy}{dx} = \frac{18x}{y(x^2+9)^2}$$

$$\frac{dy}{dx}(3, 1) = \frac{18 (3)}{(1)(3^2+9)^2} \longrightarrow \frac{dy}{dx}(3,1) = \frac{1}{6}$$

equation of tangent line to $y^2 = \frac{x^2 - 9}{x^2 + 9}$ at poit (3,1)

$$y - 1 = \frac{1}{6} (x - 3)$$

5. $\mathbf{x^{2/3} + y^{2/3} = 5 \ (1, 8)}$

$$\frac{2}{3} x^{-\frac{1}{3}} + \frac{2}{3} y^{-\frac{1}{3}} \frac{dy}{dx} = 0$$

$$\frac{2}{3 x^{1/3}} + \frac{2}{3 y^{1/3}} \frac{dy}{dx} = 0$$

$$\frac{2}{3 y^{1/3}} \frac{dy}{dx} = - \frac{2}{3 x^{1/3}}$$

$$\frac{dy}{dx} = - \frac{2}{3x^{1/3}} \bullet \frac{3y^{1/3}}{2}$$

$$\frac{dy}{dx} = - \frac{y^{1/3}}{x^{1/3}} \longrightarrow \frac{dy}{dx} = - \left(\frac{y}{x}\right)^{1/3}$$

$$\frac{dy}{dx}(1, 8) = - \left(\frac{8}{1}\right)^{1/3} \longrightarrow \frac{dy}{dx}(1, 8) = - 2$$

equation of tangent line to $x^{2/3} + y^{2/3} = 5$ at poit (1,8)

$$y-8 = -2 (x-1)$$

SOLUTIONS

6. $y^2 = x^3$

$$2y\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

$$\frac{d^2y}{dx^2} = \frac{2y\,[6x] - 3x^2\,[2\,\frac{dy}{dx}]}{(2y)^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - 6x^2\,\frac{dy}{dx}}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - 6x^2\left(\frac{3x^2}{2y}\right)}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy - \frac{9x^4}{y}}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{12xy^2 - 9x^4}{y}\right)}{4y^2}$$

$$\frac{d^2y}{dx^2} = \frac{12xy^2 - 9x^4}{4y^3}$$

7. $\ln(x+y) = x$

$$\frac{(1 + \frac{dy}{dx})}{(x+y)} = 1$$

$$1 + \frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} = x + y - 1$$

$$\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = 1 + (x + y - 1)$$

$$\frac{d^2y}{dx^2} = x + y$$