

Are you ready for Calculus?

Part A - Algebraic Manipulation: Simplify the expression

1. $\sqrt{x}(2x^2 + 3(x - (2x + 1)))$

$$2x^{\frac{5}{2}} - 3x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$$

or

$$2\sqrt{x^5} - 3\sqrt{x^3} - 3\sqrt{x}$$

3. $\frac{9x^2 - 6x + 4}{9x^2 - 1} \cdot \frac{3x^2 + 13x + 4}{27x^3 + 8}$

$$\frac{x+4}{(3x-1)(3x+2)}$$

2. $\frac{\sqrt{x}}{\sqrt{x+3}}$

$$\frac{\sqrt{x}(\sqrt{x}-3)}{x-9} \quad \text{or} \quad \frac{x-3\sqrt{x}}{x-9}$$

4. $\frac{x + \frac{1}{y}}{\frac{y}{x}}$

$$\frac{x}{y}$$

Part B - Algebraic Equations: Solve

5. Solve for y : $4x^3 + 2x^2y + xy = 5y + x$

$$y = \frac{x - 4x^3}{2x^2 + xy - 5}$$

6. Solve for x : $\frac{x+1}{3x-1} + \frac{2x+1}{3x-2} = -1$

$$x = -\frac{1}{9} \quad \text{or} \quad x = \frac{1}{2}$$

7. Solve for x : $x^4 - 13x^2 + 37 = 7$

$$x = \pm\sqrt{3} \quad \text{or} \quad x = \pm\sqrt{10}$$

8. Solve for x : $x^3 - 4x^2 + x + 6 = 0$

$$x = -1, \quad x = 3, \quad x = 2$$

Part C - Functions and Graphs:

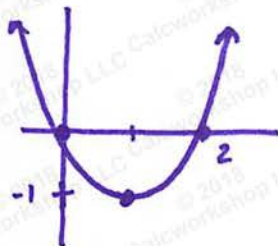
9. If $f(x) = \frac{x}{1-x}$, find $f\left(\frac{1}{x}\right)$

$$f\left(\frac{1}{x}\right) = \frac{1}{x-1}$$

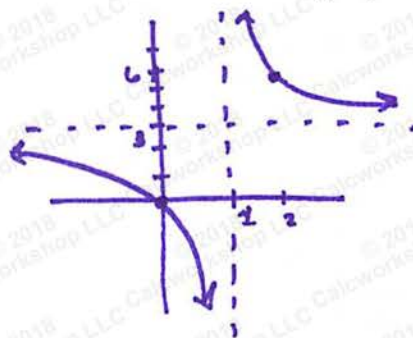
10. If $f(x) = x^2 - x - 1$, find $f(1-x)$

$$f(1-x) = x^2 - x - 1$$

11. Sketch the graph of $y = x^2 - 2x$



12. Sketch the graph of $y = \frac{3x}{x-1}$



Part D – Exponents and Logarithms:

13. Solve for x : $\log_3(x-1) = 2$

$$x = 10$$

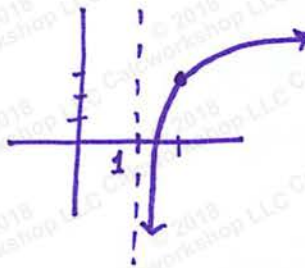
14. Solve for x : $81^x \cdot 3^{x+1} = \frac{1}{9}$

$$x = -\frac{3}{5}$$

15. Evaluate: $2 \log_2 4 + \frac{1}{2} \log_2 5 - \frac{1}{2} \log_2 20$

$$3$$

16. Graph: $y = \ln(x-1) + 3$



Part E – Trigonometry:

17. If $0 \leq \theta \leq \frac{\pi}{2}$ and $\sin \theta = \frac{12}{13}$, find $\cos \theta$

$$\cos \theta = \frac{5}{13}$$

18. Find $\sin\left(\frac{7\pi}{6}\right)$

$$-\frac{1}{2}$$

19. $\tan\left(-\frac{5\pi}{3}\right)$

$$\sqrt{3}$$

20. Solve: $\sin 2\theta - \cos \theta = 0$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

21. Solve: $-2\cos^2 x + 2\sin^2 x = 2\cos x$

$$x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

Part F – Multi-Step Question

Given the function f defined by $f(x) = 3x^3 - 2x^2 - 12x + 8$

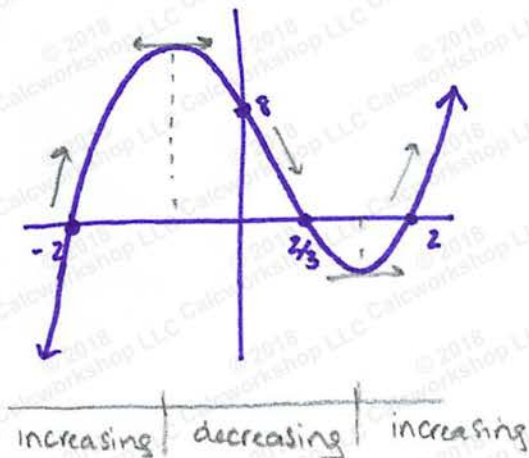
- Find the zeros of f
- Sketch the graph
- Roughly label increasing and decreasing intervals on the graph
- Find the x coordinate of all points on the graph of f where the line tangent to the graph is parallel to the x -axis. Use the fact that the slope of f is defined as $9x^2 - 4x - 12$

a) $3x^3 - 2x^2 - 12x + 8 = 0$

$$x = 2, -2, \frac{2}{3}$$

$$(2, 0), (-2, 0), \left(\frac{2}{3}, 0\right)$$

b and c)



d) parallel = same slope

$$9x^2 - 4x - 12 = 0$$

$$x = \frac{4 \pm 8\sqrt{7}}{18}$$

$$x = 1.398, -0.953$$

